1. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of $\left(2 - \frac{x}{2}\right)^7$, giving each term in its simplest form.

(4)

(b) Explain how you would use your expansion to give an estimate for the value of 1.995^7

(1)

(a) $(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \quad (n \in \mathbb{N})$

From formula booklet

So, $\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6 \cdot \left(-\frac{x}{2}\right) + \binom{7}{2}2^5 \cdot \left(-\frac{x}{2}\right)^2 + \ldots$

$= 128 + (7)(64)\left(-\frac{x}{2}\right) + (21)(32)\left(\frac{x^2}{4}\right) + \ldots$

$= 128 + 448\left(-\frac{x}{2}\right) + 672\left(\frac{x^2}{4}\right) + \ldots$

$= 128 - \frac{448x}{2} + \frac{672x^2}{4} + \ldots$

$= \boxed{128 - 224x + 168x^2 + \ldots}$

(b) $\left(2 - \frac{x}{2}\right) = 1.995$

$\frac{x}{2} = 0.005$

$\underline{x = 0.01}$

To estimate a value for 1.995^7, you would substitute 0.01 for x into the expansion

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2. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{16}\right)^9$$

giving each term in its simplest form.

(4)

$$f(x) = (a + bx)\left(2 - \frac{x}{16}\right)^9, \text{ where } a \text{ and } b \text{ are constants}$$

Given that the first two terms, in ascending powers of x, in the series expansion of $f(x)$ are 128 and $36x$,

(b) find the value of a,

(2)

(c) find the value of b.

(2)

a) $\left(2-\frac{x}{16}\right)^9 = 2^9 + 9(2)^8\left(-\frac{x}{16}\right) + 36(2)^7\left(-\frac{x}{16}\right)^2 \ldots$

$= 512 - 144x + 18x^2, \ldots$

b) $a(512) = 128$

$a = \frac{1}{4}$

$b(512) + a(-144) = 36$

$512b = 36 + 144\left(\frac{1}{4}\right)$

$= 72$

$b = \frac{9}{64}$

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3. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 + \frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

a) $\left(2+\frac{3x}{4}\right)^6 \simeq 2^6 + \binom{6}{1}(2)^5\left(\frac{3x}{4}\right)^1 + \binom{6}{2}(2)^4\left(\frac{3x}{4}\right)^2$

$\simeq \boxed{64 + 144x + 135x^2}$

b) solve $2+\frac{3x}{4} = 1.925$

and substitute the solution into our part (a) answer.

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4. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

a) use binomial formula: $(x+y)^n = \sum_0^n \binom{n}{k} x^k y^{n-k}$

$$(1+kx)^{10} = \sum_0^{10} \binom{10}{t} (kx)^t \times 1^{10-t} = \underbrace{1 + \binom{10}{1}(kx)^1 + \binom{10}{2}(kx)^2 + \binom{10}{3}(kx)^3}_{1^{st}\ 4\ \text{terms}}$$

$$= 1 + 10kx + 45k^2x^2 + 120k^3x^3$$

b) x^3 coeff. $= 3 \times x$ coeff.

$$120k^3 = 3 \times 10k$$
$$= 30k$$

$k \neq 0 \Rightarrow$ divide by k: $120k^2 = 30$

$$k^2 = \frac{1}{4}$$

$$\therefore k = \pm \frac{1}{2}$$

5. $$g(x) = (2 + ax)^8 \qquad \text{where } a \text{ is a constant}$$

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

(a) $3402x^5 = {}^8C_5 \times 2^3 \times a^5 x^5$

$= 448a^5x^5$ (1)

$3402 = 448a^5$ (1)

$a^5 = \frac{3402}{448}$

$= \frac{243}{32}$

$a = \sqrt[5]{\frac{243}{32}}$

$a = \frac{3}{2}$ # (1)

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Question continued

(b) $\left(1+\frac{1}{x^4}\right)\left(2+\frac{3x}{2}\right)^8$

$\left(1+\frac{1}{x^4}\right)\left\{\binom{8}{0}(2)^8\left(\frac{3x}{2}\right)^0+\binom{8}{1}(2)^7\left(\frac{3x}{2}\right)^1+\ldots+\right.$ (1)

$\left.\binom{8}{4}(2)^4\left(\frac{3x}{2}\right)^4+\ldots\right\}$

Constant term $= 1\times\binom{8}{0}(2^8)\left(\frac{3x}{2}\right)^0+\frac{1}{x^4}\times\binom{8}{4}(2)^4\left(\frac{3x}{2}\right)^4$

$= 1\times1\times256\times1+\frac{1}{\cancel{x^4}}\times70\times16\times\frac{81\cancel{x^4}}{16}$ (1)

$= 256+5670$

$= 5926$ (1)

(Total for Question is 7 marks)

6.

In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

Formula : k^{th} term of $(x+y)^n = \binom{n}{k} . x^k y^{n-k}$

$\Rightarrow \binom{7}{4}(2x)^4 a^3$ ① $\Rightarrow \binom{7}{4} . 2^4 \cdot a^3 = 15120$

$\Rightarrow 560a^3 = 15120$ ①

$\Rightarrow a = \sqrt[3]{\frac{15120}{560}} = 3 \Rightarrow \underline{a = 3}$ ①